A Way to Understand the Mass Generation

W-Y. Pauchy Hwang[1]
Asia Pacific Organization/Center for Cosmology and Particle Astrophysics,
Institute of Astrophysics, Center for Theoretical Sciences,
and Department of Physics, National Taiwan University, Taipei 106, Taiwan

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Abstract

We explain how the "maximally broken" family gauge theory may work; that is, the family gauge symmetry is respected at the Lagrangian level but broken spontaneously - also as a way to understand the mass-generation mechanism. We use the language of Hwang and Yan to write down an extended Standard Model - using renormalizable quantum field theory as the framework; to start with certain basic units together with a certain gauge group. Specifically we use the left-handed and right-handed spinors to form the basic units together with $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ as the gauge group. As shown in this paper, the scalar fields $\Phi(1,2)$ (the standard Higgs), $\Phi(3,1)$, and $\Phi(3,2)$ (mainly the "project-out" neutral sector, as seen in the U-gauge), with the first family index and the second $SU_L(2)$ index, would do the job - that is, to make certain that all family particles are (very) massive and the phenomena of three generations, including neutrino oscillations, are there, and nothing more.

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To propose[1] $((\nu_\tau,\,\tau)_L,\,(\nu_\mu,\,\mu)_L,\,(\nu_e,\,e)_L)$ (*columns*) ($\equiv \Psi(3,2)$) as the $SU_f(3)$ triplet and $SU_L(2)$ doublet, we immediately face *three* basic objections. Namely, the mass of the tau lepton is $1,778\,MeV$, that of the muon $105.66\,MeV$, and of the electron $0.511\,MeV$, so far apart in scale. On other hand, the charge part of the corresponding scalar fields $\Phi(3,2)$, which cannot experience the spontaneous symmetry breaking (SSB) to avoid the Goldstone boson(s), does not seem to belong at all. Thirdly, we try to propose the $SU_f(3)$ family gauge theory to understand why there are three generations - it requires all additional particles, i.e., gauge bosons and residual family Higgs, very massive. The proposal of Hwang and Yan [1] seems to be just doing the opposite.

This short note is to try to explain why - maybe our explanations would shed light on the mass generation mechanism. It may be extremely difficult to understand the *origin* of the masses. On the other hand, if we have a reasonable mathematical framework, we may use it to describe the relations among the masses - that is our humble way to proceed. The framework which we use is the *renormalizable* quantum field theory (r-QFT).

Let us recite briefly the "language" to make our presentations to be clear regarding what we would like to say.

Usually in a textbook [2], the QCD chapter precedes the one on Glashow-Weinberg-Salam (GWS) electroweak theory. Nothing is wrong with it but the basic units (or the building blocks) are further divided into the left-handed and right-handed components. It

[1]Correspondence Author; Email: wyhwang@phys.ntu.edu.tw; arXiv:1301.6464v2 [hep-ph] 11 Mar 2013.

would be nice (in helping us in thinking) if the framework is formulated all at once [1] - in an extended Standard Model we could see everything consistent with one another. Then, the questions which we pose could have broader meanings and implications.

We shall work with the Lie group $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ as the gauge group. Thus, the basic units are made up from quarks (of six flavors, of three colors, and of the two helicities) and leptons (of three generations and of the two helicities), together with all originally massless gauge bosons and the somewhat hidden induced Higgs fields. In view of the search over the last forty years, we could assume "minimum Higgs hypothesis" as the working rule.

If we look at the basic units as compared to the original particle, i.e. the electron, the starting basic units are all "point-like" Dirac particles. Dirac invented Dirac electrons eighty years ago and surprisingly enough these "point-like" Dirac particles are the basic units of the Standard Model. Thus, we call it "Dirac Similarity Principle" - a salute to Dirac; a triumph to mathematics. Our world could indeed be described by the proper mathematics. The proper mathematics may be the renormalizable quantum field theory, although our confidence in it sort of fluctuates in time.

There is no way to "prove" the above two working rules - "Dirac Similarity Principle" and "minimum Higgs hypothesis". It might be associated with the peculiar property of our Lorentz-invariant space-time. To use these two working rules, we could simplify tremendously the searches for the new extended Standard Models.

So far, we have decided on the basic units - those left-handed and right-handed quarks and leptons; the gauge group is chosen to be $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$.

In the gauge sector, the lagrangian is fixed if the gauge group is given; only for a massive gauge theory, Higgs fields are called for and we postpone its discussions until we have spelled out the fermion sector.

For the fermion sector, the story is again fixed if the so-called "gauge-invariant derivative", i.e. D_μ in the kinetic-energy term $-\bar{\Psi}\gamma_\mu D_\mu \Psi$, is given for a given basic unit [2].

Thus, we have, for the up-type right-handed quarks u_R, c_R, and t_R,

$$D_\mu = \partial_\mu - i g_c \frac{\lambda^a}{2} G^a_\mu - i\frac{2}{3} g' B_\mu, \tag{1}$$

and, for the rotated down-type right-handed quarks d'_R, s'_R, and b'_R,

$$D_\mu = \partial_\mu - i g_c \frac{\lambda^a}{2} G^a_\mu - i(-\frac{1}{3}) g' B_\mu. \tag{2}$$

On the other hand, we have, for the $SU_L(2)$ quark doublets,

$$D_\mu = \partial_\mu - i g_c \frac{\lambda^a}{2} G^a_\mu - i g \frac{\vec{\tau}}{2} \cdot \vec{A}_\mu - i\frac{1}{6} g' B_\mu. \tag{3}$$

For the lepton side, we introduce the family triplet, $(\nu^R_\tau, \nu^R_\mu, , \nu^R_e)$ (column), under $SU_f(3)$. Since the minimal Standard Model does not see the right-handed neutrinos, it would be a natural way to make an extension of the minimal Standard Model. We propose that neutrinos are only species seeing the family gauge sector. Or, we have, for $(\nu^R_\tau, \nu^R_\mu, \nu^R_e)$,

$$D_\mu = \partial_\mu - i\kappa \frac{\bar{\lambda}^a}{2} F^a_\mu. \tag{4}$$

and, for the left-handed $SU_f(3)$-triplet and $SU_L(2)$-doublet $((\nu_\tau^L,\, \tau^L),\, (\nu_\mu^L,\, \mu^L),\, (\nu_e^L,\, e^L))$ (all columns),

$$D_\mu = \partial_\mu - i\kappa \frac{\bar{\lambda}^a}{2} F_\mu^a - ig\frac{\vec{\tau}}{2} \cdot \vec{A}_\mu + i\frac{1}{2} g' B_\mu. \tag{5}$$

The right-handed charged leptons are singlets under $SU_f(3)$, or the same as in the minimum Standard Model.

The Higgs mechanism in the minimal Standard Model remains the same. For the family gauge theory, we still hope to maintain [3] that all gauge bosons be massive, i.e. $\geq$ a few TeV.

As slightly differing from the previous effort [3], we would like to write down the $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ Standard Model *all at once*, the mass term becomes

$$i\frac{\eta}{2} \bar{\Psi}^{R,T}(3,1) \times \Psi^L(3,2) \cdot \Phi(3,2) + \frac{\eta'}{2} \bar{\Psi}^R(\bar{3},1) \Psi^L(3,2) \Phi(1,2) + h.c., \tag{6}$$

where $\Psi(3,i)$ are the lepton multiplets just mentioned above (with the first number for $SU_f(3)$ and the second for $SU_L(2)$). The cross (curl) product is somewhat new [3], referring to the singlet combination of three triplets in $SU(3)$. The Higgs field $\Phi(3,2)$ is new in this effort, because it carries some nontrivial $SU_L(2)$ charge.

We add "T" explicitly to indicate the transpose in the $SU_f(3)$ space. By the "minimum Higgs hypothesis", the first coupling involves two spaces, the (3, 1) and (3, 2) internal spaces, which "should" be much smaller than, e.g., g, the interactions in the same space. By the same token, the second term involves three internal spaces; that is, it should be much further down compared to the first term. Note that the first term involves the singlet combination of three triplets - suitable for $SU(3)$; *not* an ordinary matrix operation. In this note, we are careful enough to distinguish the anti-triplet from the triplet and to realize that there are a lot of "conjugates", such as "complex conjugate", "Dirac adjoint", and "anti-triplet" (though sometime the same).

On the two terms, the first one serves as the operator suitable for neutrino oscillations while the second one is the ordinary (diagonal) mass term, but maybe smaller than radiative mass corrections induced by familons (those from Eqs. (4), (5), etc.).

The Possible Solution(s):

We return to solve the question related to the three objections raised at the beginning of this note. We do see the manifestations of the family symmetry, as in terms of three generations, even though the masses of charged leptons are widely apart. In what follows, we address the possibility that the three neutral Higgs sectors would play the (standard and family) Higgs-mechanism roles as expected to do.

Since we are dealing with a "badly broken" family gauge theory, henceforth referred to as the "maximally broken" family gauge theory, we may imagine that, in the U-gauge, the standard-model Higgs $\Phi(1,2)$ looks like $(0,\, (v+\eta(x))/\sqrt{2})$ (*column*) and $\Phi(3,2)\Phi(1,2)$ would pick out the neutral sector naturally. In fact, the term $(\Phi^\dagger(\bar{3},2)\Phi(1,2))(\Phi^\dagger(1,2)\Phi(3,2))$ with a suitable sign, would modify a massive $\Phi(3,2)$ field such that the neutral sector has SSB while the charged sector remains massive. This "project-out-Higgs" mechanism is what we are looking for.

Thus, the complex triplet $\Phi(3,1)$ and the "project-out" complex neutral part from $\Phi(3,2)$ now perform the desired Higgs mechanism - six complex fields becoming four real fields plus eight family gauge bosons [3].

Let us write down the terms for potentials among the three Higgs fields, subject to (1) that they are renormalizable, and (2) that symmetries are only broken spontaneously (the Higgs or induced Higgs mechanism).

Apart from the Higgs mechanism in the minimal Standard Model, we write

$$
\begin{aligned}
V = & \quad V_{SM} + V_1 + V_2 + V_3, & (7)\\
V_1 = & \quad \frac{M^2}{2}\Phi^\dagger(\bar{3},2)\Phi(3,2) + \frac{\lambda_1}{4}(\Phi^\dagger(\bar{3},2)\Phi(3,2))^2 & \\
+ & \quad \epsilon_1(\Phi^\dagger(\bar{3},2)\Phi(3,2))(\Phi^\dagger(1,2)\Phi(1,2)) & \\
+ & \quad \epsilon_2(\Phi^\dagger(\bar{3},2)\Phi(3,2))(\Phi^\dagger(\bar{3},1)\Phi(3,1)) + other.combinations, & (8)\\
V_2 = & \quad \frac{\mu_2^2}{2}\Phi^\dagger(\bar{3},1)\Phi(3,1) + \frac{\lambda_2}{4}(\Phi^\dagger(\bar{3},1)\Phi(3,1))^2 + \delta(\Phi^\dagger(3,1)\cdot\Phi(3,1)\times\Phi(3,1) + c.c.) & \\
+ & \quad \lambda_2'\Phi^\dagger(\bar{3},1)\Phi(3,1)\Phi^\dagger(1,2)\Phi(1,2), & (9)\\
V_3 = & \quad \eta_1(\Phi^\dagger(\bar{3},2)\Phi(1,2))(\Phi^\dagger(1,2)\Phi(3,2)) + \eta_3\Phi^\dagger(3,2)\cdot\Phi(3,2)\times(\Phi^\dagger(1,2)\Phi(3,2)) & \\
+ & \quad \nu_1(\Phi^\dagger(\bar{3},2)\Phi(1,2))\cdot\Phi^\dagger(\bar{3},2)\times\Phi(\bar{3},2) & \\
+ & \quad \nu_2(\Phi^\dagger(\bar{3},2)\Phi(1,2))\cdot\Phi^\dagger(\bar{3},1)\times\Phi(\bar{3},1) + \eta_2(\Phi^\dagger(\bar{3},2)\Phi(1,2)\Phi(3,1) + c.c.). & (10)
\end{aligned}
$$

So, at the Lagrangian level, the $SU_c(3) \times SU_L(2) \times U(1) \times SU_f(3)$ gauge symmetry is protected but is violated via spontaneous symmetry breaking (via the Higgs mechanism). The neutral sector in $\Phi(3,2)$ gets picked out, in the U-gauge, by the first term in V_3. Here we note that the "project-out" neutral sector happens everywhere in V_3, except the term in ν_1. The "induced SSB", through the Higgs mechanism in the minimal Standard Model, is an interesting new phenomenon. Regarding the signs, we have $\mu_2 < 0$ and $\lambda_2 > 0$ to guarantee the spontaneous symmetry breaking (SSB) in the $\Phi(3,1)$ sector. Further, a remarkable phenomenon happens in a natural way: $M > 0$ and $\lambda_1 >$ to ensure that there is no SSB in the charged sector while $\eta_1 v^2 + M^2 < 0$ (in the simplest case) to ensure that there is SSB in the neutral sector.

The scenario for the masses of the family particles might be as follows: For those eight familons (or family gauge bosons), we could assume a few TeV or slightly more. For those four family Higgs (those participating Higgs mechanisms), maybe even slightly more heavier. We don't have definitive expectations for the charged scalar particles, except that they could be much heavier. These are the so-called "dark-matter particles", and the experimental detection of the mass of an individual "dark-matter" particle is next to impossible.

As shown earlier [3, 4], two triplets of complex scalar fields would make the eight family gauge bosons and four residual family Higgs particles all massive, presumably heavier that a few TeV. In the $SU_f(3)$ theory alone, there are many ways of accomplishing it. In the present case, the equivalence between two triplets is lost, but in a minor way. There are so many terms in the Higgs potentials, Eqs. (7)-(10), such that the search of the detailed solution(s) should not be much affected.

End of the Proof.

In fact, the mathematics of the three neutral Higgs, $\Phi(1,2)$ (standard Higgs), $\Phi(3,1)$

(purely family Higgs), and $\Phi^0(3,2)$, subject to the renormalizabilty (up to the fourth power), turns to be very rich. In our earlier work regarding the "colored Higgs mechanism" [4], we show how the eight gauge bosons in the $SU(3)$ gauge theory become massive using two complex scalar triplet fields (with the resultant four real Higgs fields), *with a lot of choices.* It is very interesting that the choices are still there (for this paper), even though the equivalence between two complex triplets is lost. Clearly, this opens up a new broad branch of Higgs physics. Hopefully, it will bring us to a deeper understanding of the mass generation mechanism.

Neutrinos have tiny masses far smaller than the masses of the quarks or of charged leptons. Neutrinos oscillate among themselves, giving rise to a lepton-flavor violation (LFV). There are other oscillation stories, such as the oscillation in the $K^0 - \bar{K}^0$ system, but there is a fundamental "intrinsic" difference here - the $K^0 - \bar{K}^0$ system is composite while neutrinos are "point-like" Dirac particles. It is true that neutrino masses and neutrino oscillations may be regarded as one of the most important experimental facts over the last thirty years [5].

In fact, certain LFV processes such as $\mu \to e + \gamma$ [5] and $\mu + A \to A^* + e$ are closely related to the most cited picture of neutrino oscillations so far [5]. In recent publications [6], it was pointed out that the cross-generation or off-diagonal neutrino-Higgs interaction may serve as the detailed mechanism of neutrino oscillations, with some vacuum expectation value of the new Higgs fields, $\Phi(3,1)$ and $\Phi^0(3,2)$. *So, even though we haven't seen, directly, the family gauge bosons and family Higgs particles, we already see the manifestations of their vacuum expectation values.*

In the other words, the first term in the last equation [Eq. (6)] can be used as the basis to analyze the various lepton-flavor-violating decays and reactions.

To close this note, We would like to ask what the dark-matter world look like, if the extended Standard Model discussed in this paper turns out to be true.

In a slightly different context [7], it was proposed that we could work with two working rules: "Dirac similarity principle", based on eighty years of experience, and "minimum Higgs hypothesis", from the last forty years of experience. Using these two working rules, the extended model mentioned above becomes rather unique - so, it is so much easier to check it against the experiments.

We would be curious about how the dark-matter world looks like, though it is difficult to verify experimentally. The first question would be: The dark-matter world, 25 % of the current Universe (in comparison, only 5 % in the ordinary matter), would clusterize to form the dark-matter galaxies, maybe even before the ordinary-matter galaxies. The dark-matter galaxies would then play the hosts of (visible) ordinary-matter galaxies, like our own galaxy, the Milky Way. Note that a dark-matter galaxy is by our definition a galaxy that does not possess any ordinary strong and electromagnetic interactions (with our visible ordinary-matter world). This fundamental question deserves some thoughts, for the structural formation of our Universe.

Of course, we should remind ourselves that, in our ordinary-matter world, those quarks can aggregate in no time, to hadrons, including nuclei, and the electrons serve to neutralize the charges also in no time. Then atoms, molecules, complex molecules, and so on. These serve as the seeds for the clusters, and then stars, and then galaxies, maybe in a time span of $1\,Gyr$ (i.e., the age of our young Universe). The aggregation caused by strong and

electromagnetic forces is fast enough to help giving rise to galaxies in a time span of $1\,Gyr$. On the other hand, the seeded clusterings might proceed with abundance of extra-heavy dark-matter particles such as familons and family Higgs, all greater than a few TeV and with relatively long lifetimes (owing to very limited decay channels). So, further simulations on galactic formation and evolution may yield clues on our problem.

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